Problem 1: Power in a coaxial cable

A long coaxial cable consists of an inner cylinder of radius R_i and an outer cylinder of radius R_o. There is a current I flowing along the surface of the inner cylinder, and a current $-I$ (i.e., the same amount of current in the opposite direction) flowing along the surface of the outer cylinder.

(a) Use Ampère's law to determine the magnetic field in the region $R_i < s < R_o$ between the cylinders.

(b) Think of the inner current as a line charge moving at constant velocity: $I = \lambda\, v$. Use Gauss' law to determine the electric field in the region between the cylinders.

(Remember, $\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0$ even when the charges are moving, so Gauss' law works the same as before. Find the electric field for a long line charge λ.)

(c) Calculate the Poynting vector and use it to determine the energy per unit time transported down (along the direction of) the cable.

(d) Since you know the electric field you can find the potential difference $\Delta V = V_i - V_o$ by integrating $d\vec{\ell} \cdot \vec{E}$ along a line from the inner cylinder ($s = R_i$) to the outer cylinder ($s = R_o$). Use this to show that the power you computed in the last part is $P = I\,\Delta V$.

Problem 2: A monochromatic plane wave

A monochromatic plane wave with amplitude E_0 and angular frequency ω is moving in the direction that points from the origin $(0, 0, 0)$ to $(3, -4, 5)$. The polarization of the wave lies in the x-z plane. Determine the components of the (real) electric and magnetic fields for this plane wave (assume that the phase shift of the wave is zero), and calculate the Poynting vector $\vec{S}$.

Problem 3: A monochromatic spherical wave

Many sources of electromagnetic waves – stars and light bulbs, for example – radiate in all directions. A simple example of the electric field for a monochromatic electromagnetic wave produced by a spherical source is

$$\vec{E}(r, \theta, \phi, t) = A\,\frac{\sin\theta}{r}\left(\cos(kr - \omega t) - \frac{1}{kr}\sin(kr - \omega t)\right)\hat{\phi}\ , \tag{1}$$

where A is a constant and $k = \omega/c$.

(a) Use one of Maxwell's equations to determine the associated magnetic field when there are no sources present (i.e, with $\rho = 0$ and $\vec{J} = 0$). Show that the three remaining equations are all satisfied by these fields.

(b) Calculate the Poynting vector $\vec{S}$ for this wave, then calculate the *intensity vector* $\vec{I} = \langle\vec{S}\rangle$ by averaging the Poynting vector over a full period $T = 2\pi/\omega$:

$$\langle\vec{S}\rangle = \frac{1}{T}\int_0^T dt\,\vec{S} \tag{2}$$

(c) Finally, integrate $\vec{I} \cdot d\vec{a}$ over the surface of a sphere to determine the total power of the wave. This is the average rate at which the spherical source is radiating energy.